On dissociation of heavy mesons in a hot quark-gluon plasma

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We compare two mechanisms for the dissociation of heavy mesons in an infinite quark-gluon plasma: dynamic Debye screening and multiple scattering. Using the uncertainty principle inspired by a Schrödinger-like equation, we find that the criterion $a_B \simeq 1/\mu \simeq \frac{1}{\alpha_{eff}^{1/2} T}$ with $\alpha_{eff} \equiv \alpha(N_c + \frac{N_f}{2})$ is parametrically true both for the dissociation of fast moving heavy mesons with a size a_B due to dynamic Debye screening as well as for mesons at rest in the medium. In contrast, we find that the criterion for the dissociation of heavy mesons due to uncorrelated multiple scattering is parametrically $a_B \simeq \frac{1}{[\gamma \alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}$. Therefore, multiple scattering is a more efficient mechanism for the dissociation of heavy mesons in an infinite hot plasma.

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I. INTRODUCTION

The problem of dissociation of bound states in a hot QCD medium is of great importance in heavy ion collisions as it provides evidence for the creation of the quark-gluon plasma in heavy ion collisions [1]. In order to get a better understanding of the properties of this state of matter it is necessary to establish criteria under which bound states are not allowed to exist or are broken apart. We focus on the study of two particular mechanisms which we believe to be the main causes of dissociation in the plasma: Debye screening and multiple scattering with constituents of the plasma.

In Ref.[2], the authors discussed J/ψ suppression due to Debye screening by the quark-gluon plasma and the importance of this signature to diagnose quark-gluon plasma formation in heavy ion collisions. They found that the criterion for the dissociation of a J/ψ at rest is $r_{J/\psi} = 1.61 r_D$ with $r_D \equiv \frac{1}{\sqrt{2}\mu}$, the Debye screening length. In contrast, for a heavy quark-antiquark pair moving at velocity v in an infinite strongly coupled $\mathcal{N} = 4$ SYM plasma, the AdS/CFT calculation shows that the screening length goes like $r_D(v,T) \sim r_D(0,T)/\sqrt{\gamma}$ [3]. This suggests we may expect a criterion for the dissociation of heavy quarkonia in such a strongly coupled plasma of the form $r_{Q\bar{Q}} \sim r_D(0,T)/\sqrt{\gamma}$. As for the dissociation due to multiple scattering with the constituents of the plasma, Ref. [4] addresses the problem of heavy meson suppression in a finite dense QCD medium and predicts suppression of B-mesons comparable to that of D-mesons at transverse momenta as low as $p_T \sim 10$ GeV.

The calculation of the screening effect for mesons at rest is not enough to understand the suppression observed in the data from heavy ion collisions. It is also important to establish when a fast moving meson is broken apart due to the presence of the plasma. The latter case is the one explicitly addressed in this paper. For the screening effect, the full calculation is too complicated to be performed analytically and there is not an obvious way of getting a simple picture where we can get a good estimate based on the uncertainty principle as done in [2]. By considering the Dirac equation in light-cone coordinates and in light-cone gauge we propose a Schrödinger-like equation where we can rely on the uncertainty principle to get a sensible estimate for the criterion for dissociation of bound states. This approach requires the calculation of the effective field produced by a fast moving charge with respect to the plasma, which is done in the appropriate region of momenta where we keep terms only up to first order in $\frac{k_\perp}{k_z} \sim \gamma^{-1}$. Even though in covariant gauge the effective field in the rest frame

of the charge moving relative to the plasma with $v \simeq 1$ is highly anisotropic[5], we find that after a gauge transformation and going back to the rest frame of the plasma, the anisotropy is suppressed as inverse powers of γ compared to that due to the Lorentz contraction. We find that the criterion $a_B \simeq \frac{1}{\mu}$ is also parametrically true for fast moving heavy mesons.

For multiple scattering we get a criterion for dissociation in terms of the saturation momentum of the system Q_s, namely $a_B \simeq 1/Q_s$. For this purpose we first identify the typical time between interactions inside the meson τ_B. Then the criterion for dissociation is given by the statement that if the quarks inside the meson pick up enough transverse momentum during that period then the meson breaks up. We take as the natural scale for transverse momentum inside of the heavy meson $1/a_B$. The transverse momentum broadening is given by Q_s where the role of the lenght of the plasma is played by τ_B (assuming an infinite plasma). In the case of uncorrelated multiple scattering, the criterion $Q_s \simeq 1/a_B$ gives

$$a_B \simeq \frac{1}{[\gamma \alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}, \tag{1}$$

with $\alpha_{eff} \equiv \alpha(N_c + \frac{N_f}{2})$. It is also parametrically true for the dissociation of heavy mesons almost at rest with the plasma. Comparing this result with the criterion obtained from the screening effect we conclude that, in an infinite plasma, multiple scattering is a more efficient mechanism for the dissociation of heavy mesons.

The paper is organized as follows. In Sec. II, by analyzing the Dirac equation, we conclude that those photons with $|k_\perp| \lesssim 1/a_B$ and $|k_z| \lesssim \gamma/a_B$ are essential for the binding in the partonic language and get a Schrödinger-like equation in light-cone gauge in light-cone coordinates. We also discuss the typical time scale $\tau_B = \gamma/E_B$ in a heavy meson. In Sec. III, we calculate the effective field induced by a fast moving charge in light-cone gauge up to the first order in $\frac{k_\perp}{k_z} \sim \gamma^{-1}$ and use the uncertainty principle to estimate the criterion for the dissociation of fast moving heavy mesons. In Sec. IV, we give a parametric estimate for the dissociation of heavy mesons due to multiple scattering in an infinite hot quark-gluon plasma. In the Appendix, we compare the classical field A^μ calculated from classical electrodynamics and from QFT to illustrate the connection between the classical field and virtual photons.

II. THE DIRAC EQUATION FOR A FAST MOVING BOUND STATE

Even though the correct mathematical treatment of a relativistic two-body system can be done by means of the Bethe-Salpeter equation, we choose to use the Dirac equation instead in order to get a simpler physical picture. This approximation is valid when one of the particles involved is much heavier than the other, in which case the field produced by the heavier particle is no longer a dynamical variable and can be treated as an external field. In this framework, we can investigate the general properties of the wave function of the lighter quark and determine under which conditions it will be bound. In order to be able to treat the system perturbatively we still have to assume both masses are much larger than Λ_{QCD} with one of the masses much greater than the other. Although the formal results are only valid in this case, we believe the parametric result should be the same for the case of equal masses. We assume that the plasma is in the deconfinement phase and in the perturbative regime. In this circumstance, we can treat the problem in the quark-gluon plasma in a similar way as that in a QED plasma.

In the following we first investigate the role played by photons with momenta in different regions in the binding of a fast moving bound state and address the question of what is the approximate equation, appropriate for such a system, analogous to the Schrödinger equation for a bound state at rest. Then, we answer the question under which circumstances our analysis of the QED bound states applies to heavy mesons. We also give a brief illustration about the typical time scale $\tau_B = \gamma/E_B$ based on the perturbative definition of the wave function at the end of this section.

Since the detailed screening effect for a moving bound state is too complicated to be solved analytically [5], we try to simplify the problem and make it suitable for an intuitive understanding by analyzing the Dirac equation for a fast moving bound state and determining which photons are essential for the binding. This will allow us to make appropriate approximations to the calculation of the effective field induced by the heavier particle in the presence of the plasma. With the effective field, we still need a Schrödinger-like equation in light-cone coordinates which manifests the uncertainty principle and simplifies the spin structure in order to enable us to use a physical analysis of fast moving bound states similar to that for bound states at rest in Ref. [2]. In the following analysis, we assume that the lighter particle has a mass m_a and a charge e_a while the heavier particle has a mass m_b

and a charge e_b. The whole analysis in the following applies to heavy mesons provided the quarks are heavy enough.

A. Which photons are responsible for the binding?

In the vacuum case the field A^μ is much simpler, in certain gauges, in the rest frame of the bound state. Therefore, Let us start with this simpler case and assume that we have already solved $\Psi_{0n}(p)$, the wave function in the rest frame of the bound state, which peaks at $\vec{p}=0$ with a width $\Delta p = 1/a_B$, and $p^0 = E_n$. By boosting to the lab frame, we obtain $\Psi_n(p)$ which peaks at $\vec{p}_{0\perp} = 0$ with $\Delta p_\perp = 1/a_B$, $p_{0z} = v\gamma E_n$ with $\Delta p_z = \gamma/a_B$ and $p_0^0 = \gamma E_n$. By writing

$$\Psi_n(p) = \begin{pmatrix} \varphi(p) \\ \chi(p) \end{pmatrix}, \tag{2}$$

and inserting it into the Dirac equation in momentum space

$$(\not{p} - m_a)\,\Psi_n(p) = e_a \int \frac{d^4k}{(2\pi)^4} \left[\not{A}(k)\Psi_n(p-k)\right], \tag{3}$$

we get, in the chiral representation[6],

$$\begin{pmatrix} p\cdot\sigma\chi(p) - m_a\varphi(p) \\ p\cdot\bar{\sigma}\varphi(p) - m_a\chi(p) \end{pmatrix} = e_a \int \frac{d^4k}{(2\pi)^4} \begin{pmatrix} A(k)\cdot\sigma\chi(p-k) \\ A(k)\cdot\bar{\sigma}\varphi(p-k) \end{pmatrix}. \tag{4}$$

Given the above general properties of Ψ_n, we easily see that $A^\mu(k)$ with $|k_\perp| \lesssim 1/a_B$ and $|k_z| \lesssim \gamma/a_B$ gives the predominant contribution to the k integration in the right hand side of (4). As reviewed in the Appendix, in QED the classical field $A^\mu(vk_z, \vec{k})$ arises from virtual photons with momenta $\vec{k}$ in the wave-function of the heavier particle in the same gauge. Therefore, we conclude in the partonic language that those photons with $|k_\perp| \lesssim 1/a_B$ and $|k_z| \lesssim \gamma/a_B$ are essential for the binding of a bound state moving at velocity v.

This analysis allows us to give a qualitative picture of how the binding is affected by the presence of the plasma and under which circumstances the existence of bound states is not allowed. As will be seen in the next section, the presence of the plasma becomes manifest through an effective photon mass μ, which causes the corresponding screening effect. This effective mass provides a cut-off on the lower limit of the integration on the right hand side of (4) and, therefore, determines if the photons responsible for the binding are still available.

Following this argument, the criterion for the dissociation of bound states at rest in a plasma is $a_B \simeq 1/\mu$.

Back to the limit $v \simeq 1$, the arguments stated above show that we can focus only on the contribution to the field from photons with $k_z \sim \gamma k_\perp$. Before doing this, we need a simplified Dirac equation in light-cone coordinates which will enable us to use an uncertainty principle analysis for a fast moving bound state.

B. A Schrödinger-like equation in light-cone gauge in light-cone coordinates

In order to determine the existence of fast moving bound states in the presence of the plasma we would have to solve the Dirac equation corresponding to that system. For the present case, the Dirac equation is too complicated to be solved exactly. Nevertheless, we can get an approximate Schrödinger-like equation which will put us on familiar grounds to make order of magnitude estimates on the conditions under which bound states are allowed to exist. The major difficulty comes from the fact that in the rest frame of the charge the effective field is highly anisotropic in the limit $v \simeq 1$[5]. In contrast, as showed in Sec. III C, in the reference frame with the plasma at rest, the anisotropy of the effective field in light-cone gauge is predominately due to the Lorentz contraction and can be easily handled when expressed in light-cone coordinates. This allows us to neglect the anisotropy of other kinds and greatly simplify the problem. By using light-cone coordinates and keeping terms up to $\mathcal{O}(\alpha^2)$ we are able to derive an equation in which, by means of the uncertainty principle, we can establish necessary conditions for the existence of a bound state.

In coordinate space, the Dirac equation (4) takes the form

$$\left\{ \begin{array}{l} (p - e_a A) \cdot \sigma \chi = m_a \varphi \\ (p - e_a A) \cdot \bar{\sigma} \varphi = m_a \chi \end{array} \right. , \tag{5}$$

from which we can get two second order differential equations

$$\left\{ \begin{array}{l} (p - e_a A) \cdot \sigma (p - e_a A) \cdot \bar{\sigma} \varphi = m_a^2 \varphi \\ (p - e_a A) \cdot \bar{\sigma} (p - e_a A) \cdot \sigma \chi = m_a^2 \chi \end{array} \right. . \tag{6}$$

After some algebra we get

$$\left\{ \begin{array}{l} \left[(p - e_a A)^2 + e_a (\vec{B} + i\vec{E}) \cdot \vec{\sigma} \right] \varphi = m_a^2 \varphi \\ \left[(p - e_a A)^2 + e_a (\vec{B} - i\vec{E}) \cdot \vec{\sigma} \right] \chi = m_a^2 \chi \end{array} \right. . \tag{7}$$

In the following we only keep terms with expectation value up to $\mathcal{O}(\alpha^2)$ and neglect terms which are higher order in α. Under this assumption the second term can be ignored in Eq. (7) in both of the equations given there. We show a detailed calculation of the potential for the vacuum case in the Appendix which supports this statement. In the plasma the electric and magnetic fields are screened and are even weaker than in the vacuum case. Since the two equations are the same in this approximation, let us focus on one of them

$$(p - e_a A)^2 \varphi = m_a^2 \varphi. \tag{8}$$

Even though the dominant part of the potential is in the transverse components, the main contribution in the equation above comes from the A^- component since it is enhanced by a p^+ factor. In this way we get a Schrödinger-like equation in light-cone gauge in light-cone coordinates

$$p^- \varphi \simeq \left[\frac{p_\perp^2 + m_a^2}{p^+} + e_a A^- \right] \varphi, \tag{9}$$

where $p^\pm \equiv p^0 \pm p^3$. Here we have assumed $p_\perp$ is of order $\frac{1}{r} \sim \alpha m_a$ and then we have dropped terms which are higher order in α. In this paper, we will not solve (9) exactly, but instead we will use the uncertainty principle to estimate the existence of bound state solutions in the limit $v \simeq 1$. Inspired by (9), we start with

$$\langle p^- \rangle \simeq \frac{\langle p_\perp^2 \rangle + m_a^2}{\langle p^+ \rangle} + e_a A^-(\langle r \rangle), \tag{10}$$

where $\langle p^+ \rangle = [\langle p^{+2} \rangle - \Delta p^{+2}]^{\frac{1}{2}} \simeq \sqrt{\langle p^{+2} \rangle} \left[1 - \frac{\langle \Delta p^+ \rangle^2}{2\langle p^{+2} \rangle} \right] \simeq 2\gamma m_a \left[1 - \frac{\Delta p^{+2}}{2(2\gamma m_a)^2} \right]$, and we have taken $\sqrt{\langle p^{+2} \rangle} \simeq 2\gamma m_a$. In the case that $A^-(r)$ only depends on $r \equiv \sqrt{x_\perp^2 + \gamma^2 (x^-)^2}$, that is, the system has a generalized rotational symmetry, we may expect

$$\gamma \langle p^- \rangle \simeq \frac{\langle \vec{p}^2 \rangle}{2m_a} + \frac{m_a}{2} + e_a \gamma A^-(\langle r \rangle), \tag{11}$$

with $\vec{p} = (p_\perp, \Delta p^+/\gamma)$ and $\vec{x} = (x_\perp, \gamma x^-)$. Except for the different definitions of the 3-components of $\vec{x}$ and $\vec{p}$, the physical meaning of (11) is exactly the same as that used in the uncertainty principle analysis from the Schrödinger equation. In the vacuum case, we have $A^- = \frac{e_b}{4\pi} \frac{2(1-v)\gamma}{r}$ in light-cone gauge as calculated in the Appendix, and

$$\gamma \langle p^- \rangle \simeq \frac{\langle \vec{p}^2 \rangle}{2m_a} + \frac{m_a}{2} - \frac{\alpha}{\langle r \rangle}, \tag{12}$$

in the limit $\gamma \gg 1$. By using the uncertainty principle we have $\gamma \langle p^- \rangle \simeq \frac{m_a}{2} - \frac{\alpha^2 m_a}{2}$ and $p \simeq \frac{1}{r} \simeq \alpha m_a$. This is indeed consistent with the results obtained from boosted wave functions by keeping terms up to $\mathcal{O}(\alpha^2)$ in binding energy. Therefore, as in the limit $v \simeq 0$, we can get a Schrödinger-like equation in the limit $v \simeq 1$ which allows us to use the uncertainty principle to estimate the properties of fast moving bound states.

Before calculating the plasma effect on the effective potential, we give a quantitative estimate about how well the color Coulomb (perturbative) potential applies to the heavy mesons in the vacuum. Using the uncertainty principle with the Cornell confining potential $V(r) = Kr - \frac{\alpha C_F}{r}$[7], we have

$$p = \frac{K m_a}{p^2} + \alpha C_F m_a, \tag{13}$$

which tells us that if

$$m_a^2 \gg \frac{K}{(\alpha C_F)^3} \simeq \frac{0.2}{(\alpha C_F)^3} GeV^2, \tag{14}$$

we may neglect the non-perturbative linear potential responsible for the confinement in $V(r)$. In this case the discussion about the electromagnetic bound state given in this section is also valid for the heavy meson if we replace α with αC_F. For the charm quark, $\frac{0.2}{(\alpha C_F)^3} GeV^2 \simeq 1.3 GeV^2$ with $m_c^2 = 1.25^2 GeV^2 \simeq 1.6 GeV^2$ and $\alpha(m_c) = 0.4$. For the bottom quark, $\frac{0.2}{(\alpha C_F)^3} GeV^2 \simeq 10.5 GeV^2$ with $m_b^2 = 4.7^2 GeV^2 \simeq 22 GeV^2$ and $\alpha(m_b) = 0.2$. Even though for the charm and the bottom, equation (14) is not perfectly satisfied, as a parametric estimate in the following sections we can still take the binding energy $E_B \simeq \alpha^2 C_F^2 m_a$.

C. The intrinsic time-scale in a bound state

From a perturbative point of view, the interaction between the two quarks in a heavy meson occurs via interchange of gluons. It is possible to define a typical time within which we can neglect the interaction between them. This time scale will play an important role in establishing the appropriate criterion for dissociation due to multiple scattering.

Let us start with the perturbative definition of the wave function[8],

$$\begin{aligned} \Psi_{n\alpha}(x_a) = & \int d\sigma(x'_a) \, [S_F^a(x_a - x'_a) \not{n}(x'_a)]_{\alpha\rho} \, \Psi_{n\rho}(x'_a) \\ & - ie_a \int d^4 x'_a \, [S_F^a(x_a - x'_a) \not{A}(x'_a)]_{\alpha\rho} \, \Psi_{n\rho}(x'_a), \end{aligned} \tag{15}$$

where $d\sigma(x'_a)$ is the volume element of the closed 3-dimensional surface of a region of space time containing x_a, $n^\mu(x'_a)$ is the inward drawn unit normal vector of this surface at x'_a, A^μ

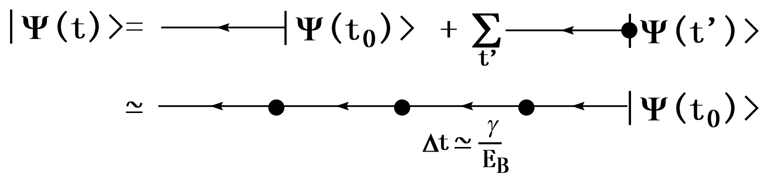

FIG. 1: The perturbative definition of a heavy meson. Black dots here represent kicks to quark a from the color field generated by quark b. If $t - t_0 \lesssim \gamma/E_B$, the wave packet propagated by the free propagator represents the predominant contribution to the wave function at t. Approximately, the picture of the interaction between the two quarks in a heavy meson is that quark a is kicked once by the color field of quark b within a period $\Delta t \lesssim \tau_B = \gamma/E_B$ to pull it back into the heavy meson.

is calculated perturbatively by the one-gluon exchange approximation and we neglect the linear potential responsible for the confinement since the masses are assumed much greater than Λ_{QCD}. Choosing the integration surface on the first term on the right hand side of (15) as the whole space at two different fixed times, we can relate the wave function at time t_a to the values of the wave function at a previous time t_0. If $\Delta t \equiv t_a - t_0 \ll \tau_B \simeq \gamma \frac{1}{\alpha^2 C_F^2 m_a}$, the first term on the right-hand side of (15) gives the predominant contribution to the wave function at t_a. Therefore, by conservation of probability, one can neglect the second term, that is, the interaction between these two quarks can be neglected. This is easy to see in light-cone coordinates, where the free propagator is [9]

$$S_F(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p^+} \left[u(p)\bar{u}(p) e^{-ip\cdot x} \Theta(x^+) - v(p)\bar{v}(p) e^{ip\cdot x} \Theta(-x^+) \right], \tag{16}$$

with $p^- = \frac{p_\perp^2 + m_a^2}{p^+}$, $\vec{p} = (p_\perp, p^+)$, and p^+ is integrated over the region $0 < p^+ < \infty$. If typically $\frac{p_\perp^2}{p^+} \Delta x^+ \sim \frac{\alpha^2 C_F^2 m_a^2}{\gamma m_a} \Delta t \simeq \Delta t / \tau_B \ll 1$, one can neglect it in the exponentials in (16) and, therefore, the integration of $\vec{x}_a'$ in the first term on the right-hand side of (15) simply gives a δ-function to reproduce the wave function at t_a in the limit $\gamma \gg 1$. Otherwise, if $\frac{p_\perp^2}{p^+} \Delta x^+ \sim \frac{\alpha^2 C_F^2 m_a^2}{\gamma m_a} \Delta t \simeq \Delta t / \tau_B \gg 1$, the first term on the right-hand side of (15) is highly suppressed due to this big term in the exponentials in the $p_\perp$ integration and, therefore, the second term on the right-hand side of (15) contributes predominately by conservation of probability. Approximately, the picture of interaction between the two quarks in such a heavy meson is that the lighter quark interacts with the color field generated by the heavier quark once every interval $\Delta t \lesssim \tau_B = \gamma/E_B$. We illustrate this in Fig. 1.

III. DYNAMIC DEBYE SCREENING EFFECT FOR FAST MOVING BOUND STATES

The next step following our analysis of the Dirac equation would be to calculate the appropriate potential for a fast moving particle to plug into equation (11). However, this whole analysis ignores completely the effect of multiple scattering with the particles in the medium, which can modify strongly the wave function of the system. Although in general these two mechanisms interfere, we will treat them separately in order to determine which one is dominant and which one should be used to establish a criterion for existence of bound states in the plasma. This approximation relies on the fact that screening is a coherent effect involving a correlated motion of particles in the medium, unlike multiple scattering which is an incoherent effect caused by random kicks from uncorrelated scattering centers. In this section, we will give a detailed calculation for the hot QED plasma, and generalize the results to the quark-gluon plasma.

A. The photon polarization vector within the HTL approximation

First, let us calculate the screening effect on the field induced by the heavy particle, due to the presence of the plasma. This is done by calculating the retarded photon propagator in thermal field theory for different regions of momenta. Following the analysis presented in the previous section, we are mainly interested in those photons with momenta $k_z \sim \gamma k_\perp$, which are essential for the binding of a fast moving bound state. In the following calculation of the retarded photon propagator, we keep terms only up to first order in $k_\perp/k_z \sim \frac{1}{\gamma}$. Moreover, we neglect the modification of A^μ due to the appearance of the lighter particle (for a detailed discussion, from the kinetic theory point of view, about the effective field A^μ in the rest frame of the particles and the influence of the appearance of another heavy particle see Ref.[5]). Even though the photon polarization vector within the hard thermal loop (HTL) approximation is well-known (say, [10, 11]), in the following we still present some details of the calculation, which enables us to see how well our approximation is in the regions of momenta beyond the HTL approximation.

Using the notation in Ref. [10], let us calculate the photon polarization vector [10, 11]

$$\begin{aligned}\Pi^{\mu\nu} &= FP_L^{\mu\nu} + GP_T^{\mu\nu} \\ &= e^2 T \sum_n \int \frac{d^3q}{(2\pi)^3} \frac{\mathrm{Tr}\left[\gamma^\mu \not{q} \gamma^\nu (\not{q} - \not{k})\right]}{[\omega_n^2 + \omega_{\vec{q}}^2][(\omega_n - \omega)^2 + \omega_{\vec{q}-\vec{k}}^2]},\end{aligned} \tag{17}$$

with $P_L^{\mu\nu} \equiv -g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} - P_T^{\mu\nu}$, $P_T^{ij} \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$, $P_T^{0\mu} = P_T^{\mu 0} = 0$ and $\omega_n = (2n+1)\pi T$. From Eq. (17), we have

$$F = \frac{\Pi^{00}}{P_L^{00}} = \frac{k^2}{\vec{k}^2} \Pi^{00}, \tag{18}$$

and

$$G = -\frac{1}{2}(F + \Pi), \tag{19}$$

with $\Pi \equiv g_{\mu\nu}\Pi^{\mu\nu}$. After taking the trace and performing the frequency sums we get,

$$\begin{aligned}\Pi^{00} = e^2 \int \frac{d^3q}{(2\pi)^3} &\left\{ \left(1 - \frac{E_1^2 - \vec{k}\cdot\vec{q}}{E_1 E_2}\right) (1 - \tilde{n}(E_1) - \tilde{n}(E_2)) \left(\frac{1}{k^0 - E_1 - E_2} - \frac{1}{k^0 + E_1 + E_2}\right) \right. \\ &\left. + \left(1 + \frac{E_1^2 - \vec{k}\cdot\vec{q}}{E_1 E_2}\right) (\tilde{n}(E_1) - \tilde{n}(E_2)) \left(\frac{1}{k^0 + E_1 - E_2} - \frac{1}{k^0 - E_1 + E_2}\right) \right\},\end{aligned} \tag{20}$$

and

$$\begin{aligned}\Pi = -2e^2 \int \frac{d^3q}{(2\pi)^3} &\left\{ \left(1 + \frac{E_1^2 - \vec{k}\cdot\vec{q}}{E_1 E_2}\right) (1 - \tilde{n}(E_1) - \tilde{n}(E_2)) \left(\frac{1}{k^0 - E_1 - E_2} - \frac{1}{k^0 + E_1 + E_2}\right) \right. \\ &\left. + \left(1 - \frac{E_1^2 - \vec{k}\cdot\vec{q}}{E_1 E_2}\right) (\tilde{n}(E_1) - \tilde{n}(E_2)) \left(\frac{1}{k^0 + E_1 - E_2} - \frac{1}{k^0 - E_1 + E_2}\right) \right\},\end{aligned} \tag{21}$$

where $\tilde{n}(E) = \frac{1}{e^{\beta E}+1}$, $E_1 = |\vec{q}|$, and $E_2 = |\vec{q} - \vec{k}|$.

In the HTL approximation, that is, $k_z, k_\perp \ll T$, we get the well-known results

$$\begin{aligned}\Pi^{00} &\simeq 2e^2 \int \frac{d^3q}{(2\pi)^3} \frac{d\tilde{n}(q)}{dq} \left(2 - \frac{k^0}{k^0 + k\cos\theta} - \frac{k^0}{k^0 - k\cos\theta}\right) \\ &= -\mu^2 \left(2 - \frac{k^0}{k} \ln \frac{\frac{k^0}{k} + 1}{\frac{k^0}{k} - 1}\right),\end{aligned} \tag{22}$$

and

$$\begin{aligned}\Pi &\simeq 4e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q} (1 - 2\tilde{n}(q)) \\ &\to -4e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q} 2\tilde{n}(q) = -2\mu^2,\end{aligned} \tag{23}$$

where we have dropped the T-independent divergent term and we have taken $\mu^2 \equiv \frac{e^2T^2}{6}$. When making use of the classical current induced by a fast moving particle, we introduce a delta function of the form $\delta(k^0 - vk_z)$, so we can safely replace $k^0 = vk_z$ in the calculation of the propagator. In our case $k_z \simeq \gamma k_\perp$, by keeping only the first order of $\frac{k_\perp}{k_z}$, we have

$$F \simeq -\mu^2 \frac{k^2}{\vec{k}^2}\left(2 - \frac{k^0}{k}\ln\frac{\frac{k^0}{k}+1}{\frac{k^0}{k}-1}\right) \simeq 2\mu^2\gamma^{-2}\ln\gamma \xrightarrow{v\to 1} 0, \tag{24}$$

and, therefore,

$$G \simeq \mu^2. \tag{25}$$

B. The photon polarization vector beyond the HTL approximation

As explained in previous sections, photons with $k_z \simeq \gamma/a_B$ play an important role in the calculation of bound state wave functions, and since we are interested in fast moving particles, k_z might be comparable with or even larger than T in the limit $v \simeq 1$. On the other hand, the transverse components of the photon momenta are not necessarily large. The relevant region for the transverse momenta is $\sim 1/a_B$ and in particular we would like to focus on the region $1/a_B \sim \alpha^{1/2}T$, which means we still have the condition $k_\perp \ll T$. In the following, we give a detailed calculation of the photon polarization vector in the limit $k_\perp \ll T$ and $k_z \gg T$.

Let us calculate F and G for this region of momenta. Since we are considering the case $|\vec{k}| \gg T$, the predominant contributions to the integrals in (20) and (21) come from the separate regions with $|\vec{q}| \sim T$ or $|\vec{q} - \vec{k}| \sim T$. Both regions give the same contribution since the integrands are unchanged under $\vec{q} \to \vec{k} - \vec{q}$. Taking into account the contribution from both regions we get

$$F = \frac{k^2}{\vec{k}^2}4e^2\int\frac{d^3q}{(2\pi)^3}\frac{\tilde{n}(q)}{q}\frac{1+\cos^2\theta}{1-\cos^2\theta} = \frac{k^2}{\vec{k}^2}\mu^2\int_0^1 d\cos\theta\frac{1+\cos^2\theta}{1-\cos^2\theta}. \tag{26}$$

The angular integration has a collinear divergence that is cutoff by the $k_\perp$ term neglected in the approximation above and by the mass of the constituents of the plasma (neglected in our calculation for high T). Therefore, as in the case $k_z, k_\perp \ll T$ in (24), this logarithmic divergence must be proportional to $\ln\gamma$, that is,

$$F \sim \gamma^{-2}\ln\gamma. \tag{27}$$

Similarly, the predominant contributions of q-integration from the (separate) regions with $|\vec{q}| \sim T$ or $|\vec{q}-\vec{k}| \sim T$ gives us

$$\Pi = -2\mu^2. \tag{28}$$

Accordingly,

$$G = \mu^2 \left(1 - \frac{1}{2}\frac{k^2}{\vec{k}^2}\int_0^1 d\cos\theta \frac{1+\cos^2\theta}{1-\cos^2\theta}\right) \simeq \mu^2. \tag{29}$$

Therefore, in the case $k_\perp \ll T$, we have $F \simeq 0$ and $G \simeq \mu^2$ in the limit $k_z \gg T$ as well as in the limit $k_z \ll T$. This justifies $F \simeq 0$ and $G \simeq \mu^2$ in the case $k_\perp \ll T$ in the following calculation even though the region with $k_z \sim T$ is difficult to evaluate analytically.

C. The effective field in light-cone gauge

To use (11) for a fast moving bound state in the plasma, we need to evaluate the retarded photon propagator in light-cone gauge. By only keeping terms up to first order in $k_\perp/k_z \simeq \gamma^{-1}$, we have

$$[D_{LC}D_{LC}]^{\mu\nu} \equiv D^{\mu}_{LC\rho}D^{\rho\nu}_{LC} \simeq \frac{-i}{k^2}D^{\mu\nu}_{LC}, \tag{30}$$

and

$$\begin{aligned}&[-iD_{LC}\Pi D_{LC}]^{\mu\nu} \equiv -iD^{\mu\rho}_{LC}\Pi_{\rho\sigma}D^{\nu\sigma}_{LC}\\ &\simeq \frac{i\mu^2}{[k^2]^2}\left[P^{\mu\nu}_T - \frac{1}{\eta\cdot k}\left(k^\mu\eta_\sigma P^{\nu\sigma}_T + k^\nu\eta_\sigma P^{\mu\sigma}_T\right) + \frac{k^\mu k^\nu}{(\eta\cdot k)^2}\eta^\rho\eta^\sigma P^T_{\rho\sigma}\right] \simeq \frac{\mu^2}{k^2}D^{\mu\nu}_{LC},\end{aligned} \tag{31}$$

where $D^{\mu\nu}_{LC}$, defined in (54), is the vacuum light-cone gauge propagator. Using (30) and (31), we get the full photon propagator

$$\begin{aligned}D^{\mu\nu}_R &= D^{\mu\nu}_{LC} + [-iD_{LC}\Pi D_{LC}]^{\mu\nu} + \left[(-i)^2 D_{LC}\Pi D_{LC}\Pi D_{LC}\right]^{\mu\nu} + \cdots\\ &\simeq -\frac{i}{k^2-\mu^2}\left[g^{\mu\nu} - \frac{\eta^\mu k^\nu + \eta^\nu k^\mu}{\eta\cdot k}\right].\end{aligned} \tag{32}$$

The corresponding effective field is given by Maxwell's equations

$$A^\mu(k) = -iD^{\mu\nu}_R(k)j_\nu \simeq 2\pi e_b\delta(\omega - vk_z)\left(\frac{v-1}{k^2-\mu^2}, \frac{\vec{k}_\perp}{k_z(k^2-\mu^2)}, \frac{1-v}{k^2-\mu^2}\right), \tag{33}$$

and in coordinate space we have

$$A^-(x) = \frac{e_b}{4\pi}2(1-v)\gamma\frac{e^{-\mu r}}{r}, \tag{34}$$

and

$$\begin{aligned}\vec{A}_\perp(x) &= \frac{i}{2}e_b\,\nabla_\perp\int\frac{d^3k}{(2\pi)^3}e^{-ik\cdot x}\frac{1}{[k_\perp^2+\gamma^{-2}k_z^2+\mu^2]}\left(\frac{1}{k_z+i\epsilon}+\frac{1}{k_z-i\epsilon}\right)\\ &\simeq i\frac{e_b}{4\pi}\left[\Theta(x^-)-\Theta(-x^-)\right]\nabla_\perp\int dk_\perp k_\perp\frac{J_0(k_\perp x_\perp)}{k_\perp^2+\mu^2}\\ &\simeq\frac{e_b}{4\pi}\frac{\vec{x}_\perp}{x_\perp^2}\mu x_\perp K_1(\mu x_\perp)\left[\Theta(x^-)-\Theta(-x^-)\right],\end{aligned}\tag{35}$$

where $r\equiv\sqrt{x_\perp^2+\gamma^2(z-vt)^2}\simeq\sqrt{x_\perp^2+\gamma^2(x^-)^2}$ and in the calculation of $A_\perp(x)$ we have only picked up the poles at $k_z=\pm i\epsilon$.

In kinetic theory, the effective field $\tilde{A}^\mu(k)$ is found to be strongly anisotropic for $v\simeq 1$ in the rest frame of the charge [5], which seems to contradict the results in light-cone gauge. However, we shall see that after some gauge transformation and going back to the rest frame of the plasma, the anisotropy is suppressed as inverse powers of γ in contrast with that due to the Lorentz contraction. In the limit $v\simeq 1$, the effective field $\tilde{A}^\mu(k)$ in the rest frame of the charge calculated in Ref. [5] is

$$\begin{aligned}\tilde{A}^0(k) &\simeq 2\pi e_b\delta(\omega)\left[\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\cos^2\theta}\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right)\right],\\ \tilde{A}^1(k) &\simeq -2\pi e_b\delta(\omega)\tan\theta\cos\phi\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right),\\ \tilde{A}^2(k) &\simeq -2\pi e_b\delta(\omega)\tan\theta\sin\phi\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right),\\ \tilde{A}^3(k) &\simeq 2\pi e_b\delta(\omega)\tan^2\theta\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right),\end{aligned}\tag{36}$$

with $k^\mu=(\omega,|\vec{k}|\cos\phi\sin\theta,|\vec{k}|\sin\phi\sin\theta,|\vec{k}|\cos\theta)$. Equ. (36) is the same as calculated in the above approximation $F\simeq 0$ and $G\simeq\mu^2$ in covariant gauge. After the gauge transformation $\tilde{A}^\mu(k)\to\tilde{A}^\mu(k)+k^\mu\Lambda(k)$ with

$$\Lambda(k)=2\pi e_b\delta(\omega)\frac{1}{|\vec{k}|\cos\theta}\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right),\tag{37}$$

we have

$$\begin{aligned}\tilde{A}^0(k) &\simeq 2\pi e_b\delta(\omega)\left[\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\cos^2\theta}\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right)\right],\\ \tilde{A}_\perp(k) &\simeq 0,\\ \tilde{A}^3(k) &\simeq 2\pi e_b\delta(\omega)\frac{1}{\cos^2\theta}\left(\frac{1}{\vec{k}^2+\mu^2}-\frac{1}{\vec{k}^2}\right).\end{aligned}\tag{38}$$

Back to the rest frame of the plasma, it gives

$$
\begin{aligned}
A^0(k) &\simeq -2\pi e_b \delta(\omega - v k_z) \frac{1}{k^2 - \mu^2}, \\
A_\perp(k) &\simeq 0, \\
A^3(k) &\simeq -2\pi e_b \delta(\omega - v k_z) \frac{1}{k^2 - \mu^2},
\end{aligned}
\tag{39}
$$

showing that the anisotropy is suppressed as inverse powers of γ in the rest frame of the plasma. We have shown that the effective field of a fast moving charge has a smooth limit as $v \to 1$. Moreover, by using light-cone gauge in the rest frame of the plasma, one can obtain an effective field in which the anisotropy is predominantly due to Lorentz contraction. This allows us to use (11) to obtain a criterion for the dissociation of fast moving heavy mesons due to dynamic Debye screening.

D. Dissociation of heavy mesons due to dynamic Debye screening

In the HTL approximation, one can obtain the gluon self-energy simply by taking $\mu^2 = \frac{1}{6}g^2T^2(N_c + \frac{N_f}{2})$ in (24) and (25)[11], where N_f is the number of massless flavors of quarks in the quark-gluon plasma. Since this result does not depend on the gauge-fixing [12], by replacing e_b with $-g$, we get the effective gluon field in light-cone gauge

$$
A^-(x) = -\frac{g}{4\pi} 2(1-v)\gamma \frac{e^{-\mu r}}{r}, \tag{40}
$$

and

$$
A_\perp(x) \simeq -\frac{g}{4\pi} \frac{\vec{x}_\perp}{x_\perp^2} \mu x_\perp K_1(\mu x_\perp) \left[\Theta(x^-) - \Theta(-x^-)\right]. \tag{41}
$$

Now, we are ready to discuss the criterion for the dissociation of fast moving heavy mesons in the quark-gluon plasma by using the uncertainty principle in light-cone gauge. Inserting (34) into (11), we have

$$
\gamma \left\langle p^- \right\rangle \simeq \frac{\vec{p}^2}{2m_a} + \frac{m_a}{2} - 2(1-v)\gamma^2 \frac{\alpha C_F e^{-\mu r}}{r} \simeq \frac{\vec{p}^2}{2m_a} + \frac{m_a}{2} - \frac{\alpha C_F e^{-\mu r}}{r}, \tag{42}
$$

in the limit $v \simeq 1$. Assuming $r = 1/p$ and minimizing (42), we have

$$
\frac{\mu}{\alpha C_F m_a} = x(1+x)e^{-x}, \tag{43}
$$

with $x \equiv \mu/p$. Eq. (43) has solutions only if $\mu a_B \equiv \frac{\mu}{\alpha C_F m_a} \leq 0.84$ (see [2]). Therefore, we can use the uncertainty principle argument for the dissociation of fast moving heavy mesons

in the plasma in the same way as it was used for bound states at rest [2]. The corresponding criterion for the dissociation of fast moving heavy mesons, based on a screening analysis, is

$$a_B \simeq 1/\mu. \tag{44}$$

Note, the anisotropy in the effective field seen in the rest frame of the charge could imply a more efficient screening than that in the case with the charge almost at rest with the plasma, but the modification of this criterion on the right hand of (44) should not depend on inverse powers of γ.

IV. DISSOCIATION OF HEAVY MESONS DUE TO MULTIPLE SCATTERING

In this section, we give a parametric estimate of the criterion for the dissociation of heavy mesons due to multiple scattering in terms of the saturation momentum Q_s, which is a characteristic property of any QCD media[13, 14]. Quantitatively, the physical meaning of Q_s is that the gluon distribution of the target is dense as seen by probes with virtuality $q_\perp \ll Q_s$, but it is dilute as seen by probes with high virtuality. When a heavy meson with a size a_B travels in the quark-gluon plasma, $1/a_B$ naturally plays the role of the virtuality $q_\perp$. If $a_B \gg 1/Q_s$, the meson will break up, that is, the medium looks opaque to the meson. This picture is confirmed by detailed calculations in Ref. [15].

This picture can also be justified by the argument of the transverse momentum broadening of the two quarks in a heavy meson. As showed in Fig. 1, within a period τ_B, the two particles in a heavy meson propagate in the medium like two free quarks and pick up the transverse momentum broadening $\langle \Delta p_\perp^2 \rangle$, which is equal to the saturation momentum squared Q_s^2[16]. If $\langle \Delta p_\perp^2 \rangle = Q_s^2 \gg 1/a_B^2$, that is, the two quarks pick up transverse momenta greater than the typical momentum in a bound state, the meson will break up. Therefore, the criterion for the dissociation of a heavy meson due to multiple scattering in a medium is parametrically $a_B \simeq 1/Q_s$. This picture applies to cold matter as well as to hot matter[15]. In the following, we shall give a parametric analysis of the criterion for the dissociation of heavy mesons in a hot quark-gluon plasma in the case that the successive scatterings are uncorrelated.

In a hot quark-gluon plasma modeled by uncorrelated scattering centers, for a fast moving quark, Q_s^2 has the following simple form[16]

$$Q_s^2 \simeq \frac{L}{\lambda}\mu^2 \ln\frac{T^2}{\mu^2} = L\rho\sigma\mu^2 \ln\frac{T^2}{\mu^2} \simeq \alpha^2 C_F (N_c + \frac{N_f}{2}) T^3 L \ln\frac{1}{\alpha_{eff}}, \tag{45}$$

where we have taken the number density $\rho \simeq (N_c + \frac{N_f}{2})T^3$, $\sigma = \frac{4\pi\alpha^2}{\mu^2}C_F$ and $\mu^2 = \frac{1}{6}(N_c + \frac{N_f}{2})g^2T^2 \simeq \alpha(N_c + \frac{N_f}{2})T^2 \equiv \alpha_{eff}T^2$. In a finite plasma with a length $L \lesssim \tau_B$, the criterion for the dissociation of heavy mesons is $\alpha^2 C_F(N_c + \frac{N_f}{2}) \ln \frac{1}{\alpha_{eff}} T^3 L \simeq \frac{1}{a_B^2}$. In an infinite quark-gluon plasma, the time scale $\tau_B = \frac{\gamma}{E_B} \simeq \frac{\gamma a_B}{\alpha C_F}$ plays the role of the length L in the definition of Q_s, and our criterion for dissociation of heavy mesons $Q_s^2 \simeq 1/a_B^2$ gives $a_B \simeq \frac{1}{[\gamma\alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}$ for the quark-gluon plasma, in contrast with that due to the screening effect $a_B \simeq \frac{1}{\mu} \simeq \frac{1}{\alpha_{eff}^{\frac{1}{2}} T}$.

If the successive scatterings between the mesons and the plasma constituents are essentially independent of each other, the criterion $a_B \simeq \frac{1}{[\gamma\alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}$ is also true for the dissociation of heavy mesons almost at rest with the plasma. Quantitatively, in a hot plasma this means the collision time $\tau_c \simeq \lambda = \frac{1}{\rho\sigma} \simeq \frac{1}{\alpha C_F T} \gg 1/\mu$, which is equivalent to $1 \sim \frac{N_c + \frac{N_f}{2}}{C_F^2} \gg \alpha$. In this case, non-relativistic quarks pick momentum broadening symmetrically in each direction due to the uncorrelated random kicks from plasma constituents in the same way as the transverse momentum broadening of relativistic quarks. Therefore, we would expect that the momentum broadening of non-relativistic quarks should have a similar form as (45).

Since uncorrelated multiple scattering implies $\alpha \ll 1$, we can use the leading-log approximation[17] in our discussion about the criterion for the dissociation of non-relativistic heavy mesons. The mean-squared momentum transfer per unit time between a non-relativistic heavy quark and the plasma is calculated in Ref. [18] and by keeping only the leading log terms, we have

$$\frac{d}{dt}\left\langle(\Delta p)^2\right\rangle \simeq \frac{4\pi}{3}\alpha^2 C_F(N_c + \frac{N_f}{2}) \ln \frac{T^2}{\mu^2} T^3. \tag{46}$$

Since Eq. (46) is time-independent,

$$\left\langle(\Delta p)^2\right\rangle \simeq \frac{4\pi}{3}\alpha^2 C_F(N_c + \frac{N_f}{2}) \ln \frac{T^2}{\mu^2} T^3 \Delta t. \tag{47}$$

Taking $\Delta t \simeq \tau_B \simeq \frac{1}{E_B}$ and $\langle(\Delta p)^2\rangle \simeq 1/a_B^2$, we get $a_B \simeq \frac{1}{[\alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}$. Therefore, we conclude that $a_B \simeq \frac{1}{[\gamma\alpha_{eff} \ln \frac{1}{\alpha_{eff}}]^{\frac{1}{3}} T}$ is a parametric criterion for the dissociation of a heavy meson due to uncorrelated multiple scattering in an infinite plasma. This criterion was also obtained in Ref.[19] in the case of heavy quarkonia at rest with the plasma by effective field theory techniques, which was first obtained (without the logarithm) in [20]. At high energies, heavy mesons can dissociate even at much lower temperature T before dynamic Debye screening effect plays an important role.

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APPENDIX: The Classical Field A^{μ}_{cl} in Coulomb and Light-cone Gauges

In this appendix, we compare the field A^{μ} for a charge e_b moving with a velocity v along the z direction in classical electrodynamics and the classical field A^{μ}_{cl} for a charged particle with $p^{\mu} = p^0(1, 0_{\perp}, v)$ and charge e_b in the partonic picture in QED. In classical electrodynamics, the field is calculated using Maxwell's equations with a classical current. In momentum space, we have

$$A^{\mu}(k) = -iD^{\mu\nu}(k) j_{\nu}(k), \tag{48}$$

and

$$j^{\mu}(k) = 2\pi e_b \delta(\omega - v k_z)\,(1, 0_{\perp}, v)\,. \tag{49}$$

(i) Coulomb gauge $\bigtriangledown \cdot \vec{A} = 0$

The photon propagator in Coulomb gauge is

$$D^{\mu\nu}_C(k) = \begin{pmatrix} \frac{i}{\vec{k}^2} & 0 \\ 0 & \frac{iP^{ij}_T}{k^2} \end{pmatrix} \tag{50}$$

with $P^{ij}_T \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$. The classical electromagnetic field is

$$A^{\mu}(k) = -iD^{\mu\nu}_C(k) j_{\nu}(k) = 2\pi e_b \delta(\omega - v k_z)(\frac{1}{\vec{k}^2}, \frac{-vP^{i3}_T}{k^2}). \tag{51}$$

In the limit $k_z \gg k_{\perp}$ and $v \simeq 1$, we have

$$P^{i_{\perp}3}_T \simeq -\frac{k^{i_{\perp}}_{\perp}}{k_z}, P^{33}_T \simeq \frac{k^2_{\perp}}{k^2_z}, \tag{52}$$

and

$$A^{\mu}(k) \simeq 2\pi e_b \delta(\omega - v k_z)(\frac{1}{k^2_z}, -\frac{\vec{k}_{\perp}}{k_z k^2_{\perp}}, \frac{1}{k^2_z}), \tag{53}$$

which has the same transverse components as (56).

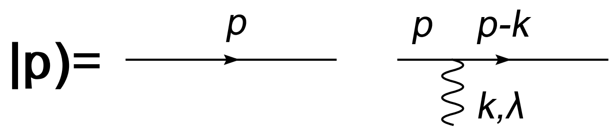

FIG. 2: The dressed wave function of a charged particle

(ii) Light-cone gauge $A^+ = 0$

The photon propagator is

$$D^{\mu\nu}_{LC}(k) = -\frac{i}{k^2}\left[g^{\mu\nu} - \frac{\eta^\mu k^\nu + \eta^\nu k^\mu}{\eta \cdot k}\right], \tag{54}$$

and

$$A^\mu(k) = -iD^{\mu\nu}_{LC}(k)j_\nu(k) = 2\pi e_b \delta(\omega - v k_z)(-\frac{1-v}{k^2}, \frac{\vec{k}_\perp}{k_z k^2}, \frac{1-v}{k^2}), \tag{55}$$

where $\eta^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)$. In the limit $v \simeq 1$, we have

$$A^\mu(k) \to 2\pi e_b \delta(\omega - v k_z)(0, -\frac{\vec{k}_\perp}{k_z k_\perp^2}, 0), \tag{56}$$

which is the same as the Weizsächer-Williams field calculated in the light-cone wave function of particle b in the partonic picture [14].

In QED, the field is quantized and virtual photons appear in the wave function of a charged particle to give rise to the classical field. In this appendix, we use the notations in Ref. [6] and the classical field is defined by[14]

$$A^\mu_{cl}(\vec{x}) = \int \frac{d^3p'}{2E_{\vec{p}'}(2\pi)^3} (p'|\, \hat{A}^\mu(\vec{x})\, |p), \tag{57}$$

where $\hat{A}^\mu(\vec{x})$ is the quantized photon field and $|p)$ is the dressed wave function of the charge particle as showed in Fig. 2

$$|p) = |p\rangle + \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} \Psi_\lambda(k)\, |p-k; k, \lambda\rangle, \tag{58}$$

with

$$\begin{aligned} 2k^0(2\pi)^3 2E_{\vec{p}-\vec{k}} \Psi_\lambda(k) \delta(\vec{p} - \vec{p}') &= \frac{\langle p'-k; k, \lambda|\, H_I\, |p\rangle}{p^0 - (p-k)^0 - k^0} \\ = e_b (2\pi)^3 \delta(\vec{p} - \vec{p}') \frac{\bar{u}(p-k) \not\epsilon^*_\lambda(k) u(p)}{p^0 - (p-k)^0 - k^0}. \end{aligned} \tag{59}$$

Inserting (58) into (57), we have

$$A^{\mu}_{cl}(\vec{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \epsilon^{\mu}_{\lambda}(k)\Psi_{\lambda}(k) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} A^{\mu}_{cl}(k). \tag{60}$$

Here, we only keep the positive-energy part of A^{μ}_{cl}.

The classical field A^{μ}_{cl} in light-cone gauge is well-known[14]. And we shall calculate it in Coulomb gauge as another example to illustrate the correspondence between A^{μ} and A^{μ}_{cl}. In Coulomb gauge,

$$\sum_{\lambda=\pm} \epsilon^{\mu}_{\lambda}\epsilon^{\nu *}_{\lambda} = P^{\mu\nu}_{T}. \tag{61}$$

Assuming $p^0 \simeq p_z \gg k^0$, we have

$$\Psi_{\lambda}(k) \simeq -\frac{e_b}{2} \frac{\epsilon^0_{\lambda} - v\epsilon^3_{\lambda}}{\vec{k}^2(1 - v\hat{k}_z)} \simeq -e_b \frac{\epsilon^0_{\lambda} - v\epsilon^3_{\lambda}}{\vec{k}^2(1 - v^2\hat{k}^2_z)} = e_b \frac{\epsilon^0_{\lambda} - v\epsilon^3_{\lambda}}{k^2}, \tag{62}$$

and

$$A^{i}_{cl}(\vec{k}) \simeq e_b \frac{P^{i0}_{T} - vP^{i3}_{T}}{k^2} \simeq e_b \frac{-vP^{i3}_{T}}{k^2}, \tag{63}$$

with $k^2 = v^2k^2_z - \vec{k}^2$, which, if multiplied by $2\pi\delta(\omega - vk_z)$, is the same as the vector potential in (51). Therefore, both calculations in Coulomb and light-cone gauges illustrate the fact that the field $A^{\mu}(\omega, \vec{k})$ with $\omega = vk_z$ in a certain gauge in classical electrodynamics arises from the virtual photon with momenta $k^{\mu} = (|\vec{k}|, \vec{k})$ in the same gauge in QED.

Since the Dirac equation is more convenient to solve in coordinate space, we shall evaluate the field $A^{\mu}(x)$ by the Fourier transformation of (51) and (55). In Coulomb gauge, $A^{\mu}(x)$ is

$$\begin{aligned} A^{\mu}(x) &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} A^{\mu}(k) \\ &= e_b \left(\frac{1}{4\pi r}, v\frac{\partial^2}{\partial\vec{x}_{\perp}\partial z}\Lambda(x), v\left(\frac{\gamma}{4\pi\tilde{r}} + \frac{\partial^2}{\partial z^2}\Lambda(x) \right) \right), \end{aligned} \tag{64}$$

where $r \equiv \sqrt{x_\perp^2 + (z-vt)^2}$, $\tilde{r} \equiv \sqrt{x_\perp^2 + \gamma^2(z-vt)^2}$ [1], and

$$\begin{aligned}
\Lambda(x) &\equiv \int \frac{d^3k}{(2\pi)^3} \frac{e^{-ik\cdot x}}{\vec{k}^2(k_\perp^2+\gamma^{-2}k_z^2)} = \int \frac{d^3k}{(2\pi)^3} \int_0^1 d\eta \frac{e^{-ik\cdot x}}{[k_\perp^2+(\eta+(1-\eta)\gamma^{-2})k_z^2]^2} \\
&= \int_0^1 d\eta \int \frac{dk_\perp dk_z k_\perp}{(2\pi)^2} \frac{e^{ik_z(z-vt)} J_0(k_\perp x_\perp)}{[k_\perp^2+(\eta+(1-\eta)\gamma^{-2})k_z^2]^2} \\
&= \int_0^1 d\eta \int \frac{dk_z}{8\pi^2} e^{ik_z(z-vt)} \frac{x_\perp K_1(\sqrt{(\eta+(1-\eta)\gamma^{-2})k_z^2 x_\perp^2})}{\sqrt{(\eta+(1-\eta)\gamma^{-2})k_z^2}} \\
&= -\int_0^1 \frac{d\eta}{8\pi} \frac{\sqrt{(\eta+(1-\eta)\gamma^{-2})x_\perp^2+(z-vt)^2}}{\eta+(1-\eta)\gamma^{-2}} \\
&= \frac{1}{4\pi} \frac{r-\gamma^{-1}\tilde{r}-(z-vt)\mathrm{ArcTanh}\left(\frac{r}{z-vt}\right)+(z-vt)\mathrm{ArcTanh}\left(\frac{\gamma^{-1}\tilde{r}}{z-vt}\right)}{\gamma^{-2}-1}.
\end{aligned} \tag{65}$$

Inserting (65) into (64), we obtain

$$A^\mu(x) = \frac{e_b}{4\pi}\left(\frac{1}{r}, \frac{1}{v}\frac{\vec{x}_\perp}{x_\perp^2}(z-vt)\left(\gamma\frac{1}{\tilde{r}}-\frac{1}{r}\right), \frac{1}{v}\left(\frac{1}{r}-\frac{1}{\gamma}\frac{1}{\tilde{r}}\right)\right). \tag{66}$$

In light-cone gauge, in coordinate space,

$$\begin{aligned}
\vec{A}_\perp(x) &= -e_b \int \frac{d^3k}{(2\pi)^3} e^{-ik\cdot x} \frac{\vec{k}_\perp}{[k_\perp^2+\gamma^{-2}k_z^2]\, k_z} \\
&= ie_b \bigtriangledown_\perp \int \frac{d^3k}{(2\pi)^3} e^{-ik\cdot x} \frac{1}{[k_\perp^2+\gamma^{-2}k_z^2]\, k_z} \\
&= ie_b \bigtriangledown_\perp \int \frac{dk_z dk_\perp k_\perp}{(2\pi)^2} e^{ik_z(z-vt)} \frac{J_0(k_\perp x_\perp)}{[k_\perp^2+\gamma^{-2}k_z^2]\, k_z} \\
&= ie_b \bigtriangledown_\perp \int \frac{dk_z}{(2\pi)^2} \frac{e^{ik_z(z-vt)}}{k_z} K_0(\sqrt{k_z^2\gamma^{-2}x_\perp^2}) \\
&= -\frac{e_b}{4\pi} \bigtriangledown_\perp \mathrm{ArcSinh}\frac{\gamma(z-vt)}{x_\perp} \\
&= \frac{e_b}{4\pi}\frac{\vec{x}_\perp}{x_\perp^2}\frac{\gamma(z-vt)}{\tilde{r}},
\end{aligned} \tag{67}$$

and

$$A^\mu(x) = \frac{e_b}{4\pi}\left((1-v)\gamma\frac{1}{\tilde{r}}, \frac{\vec{x}_\perp}{x_\perp^2}\frac{\gamma(z-vt)}{\tilde{r}}, -(1-v)\gamma\frac{1}{\tilde{r}}\right). \tag{68}$$

In the limit $v \to 1$,

$$A^\mu(x) = \frac{e_b}{4\pi}\left(0, \frac{\vec{x}_\perp}{x_\perp^2}\left[\Theta(x^-)-\Theta(-x^-)\right], 0\right), \tag{69}$$

which is the same as $A^\mu_{cl}(x)$ in Ref.[14].

[1] Note, in Sec. II and Sec. III, this is defined as r.

It is interesting to notice the big difference of the role played by different components of the classical field $A^\mu(x)$ in light-cone gauge in scattering processes at high energies and in fast moving bound systems. In a scattering process at high energies, if we take the right-mover as a classical current, since the other particle involved is a left-mover, only the transverse components $\vec{A}_\perp$ of the right-mover give a dominant contribution to the amplitude of the process[14]. On the other hand, in a fast moving bound state, since the two particles involved are both, say, right-movers, it is A^- that plays a more important role since it apears in the Dirac equation via the product with p^+ to contribute a term of order α^2 while the contribution from $\vec{A}_\perp$ with the product of $\vec{p}_\perp$ is of order α^3.

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